SIGNAL AND SYSTEMS

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MCQ FOR COMPETITVE EXAMINATIONS

SSASIT, SURAT

Continuous time signal part I

1. Which of the following signals is/are periodic?
   (a) $s(t) = \cos 2t + \cos 3t + \cos 5t$
   (b) $s(t) = \exp(j8\,\pi t)$
   (c) $s(t) = \exp(-7t) \sin 10\pi t$
   (d) $s(t) = \cos 2t \cos 4t$

[GATE 1992: 2 Marks]

Soln. (a) $s(t) = \cos 2\,t + \cos 3\,t + \cos 5\,t$

First term has $\omega_1 = 2$

Second term has $\omega_2 = 3$

Third term has $\omega_3 = 5$

Note that ratio of any two frequencies equals p/q is rational where p and q are integers.

Thus $s(t)$ is periodic

(b) $s(t) = \exp(j8\pi t)$

$= \cos(8\pi t) + j\sin(8\pi t)$

$$\frac{\omega_1}{\omega_2} = \frac{8\pi}{8\pi} = 1 \qquad so\ periodic$$

(c) $s(t) = \exp(-7t)\,.\sin 10\pi t$

$= e^{-7t}.\sin 10\pi t$

$$Note, \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = \frac{1}{5}$$

Due to e^{-7t} it is decaying function, so not periodic

(d) $s(t) = \cos 2\,t\,.\cos 4\,t$

Note, $2\cos A\cos B = \cos(A-B) + \cos(A+B)$

$so,\ s(t) = \frac{1}{2}[\cos 2\,t + \cos 6\,t]$

$$Note, \quad \frac{p}{q} = \frac{\omega_1}{\omega_2} = \frac{2}{6} = \frac{1}{3}$$

Rational

So, (a) , (b) and (d) are periodic.

2. The power in the signal

$s(t) = 8cos\left(20\pi t - \frac{\pi}{2}\right) + 4sin(15\pi t)\ is$

(a) 40
(b) 41
(c) 42
(d) 82

[GATE 2005: 1 Mark]

Soln. Time average of energy of a signal = Power of Signal

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} f^2(t)dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 dt$$

Signal power P is mean of the signal amplitude squared value of f(t) . Rms value of signal $= \sqrt{P}$

$$S(t) = 8\cos\left(20\pi t - \frac{\pi}{2}\right) + 4\sin(15\pi t)$$

$$= 8\sin(20\pi t) + 4\sin(15\pi t)$$

$$= \frac{8^2}{2} + \frac{4^2}{2} = 32 + 8 = 40$$

Option (a)

3. If a signal f(t) has energy E, the energy of the signal f(2t) is equal to

(a) E (c) 2E

(b) E/2 (d) 4E

[GATE 2001: 1 Mark]

Soln. Energy of a signal is given by

$$E = \int_{-\infty}^{\infty} [f(t)]^2 dt$$

Energy of the signal f(2t) is

$$E_s = \int_{-\infty}^{\infty} [f(t)]^2 \, dt$$

Let $2t = p \;\; or, \;\; dt = \frac{dp}{2}$

$$= \int_{-\infty}^{\infty} [f(t)]^2 \, \frac{dp}{2}$$

$$E_s = \frac{E}{2}$$

Option (b)

4. For a periodic signal
   $v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \frac{\pi}{4})$, the fundamental frequency in rad/s is

(a)	100	(c)	500
(b)	300	(d)	1500

[GATE 2013: 1 Mark]

Soln. First term has $\omega_1 = 100$

Second term $\omega_2 = 300$

Third term $\omega_3 = 500$

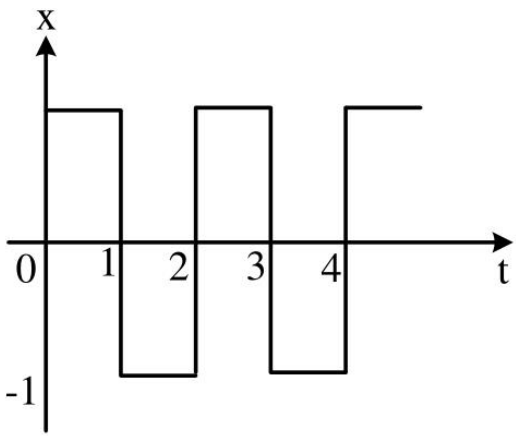
ω_1 is the fundamental frequency

ω_2 is third harmonic

ω_3 is 5th harmonic

Option (a)

5. Consider the periodic square wave in the figure shown



The ratio of the power in the 7^{th} harmonic to the power in the 5^{th} harmonic for this waveform is closest in value to ------

[GATE 2014: 1 Mark]

Soln. For a periodic square wave n^{th} harmonic component $\propto \frac{1}{n}$

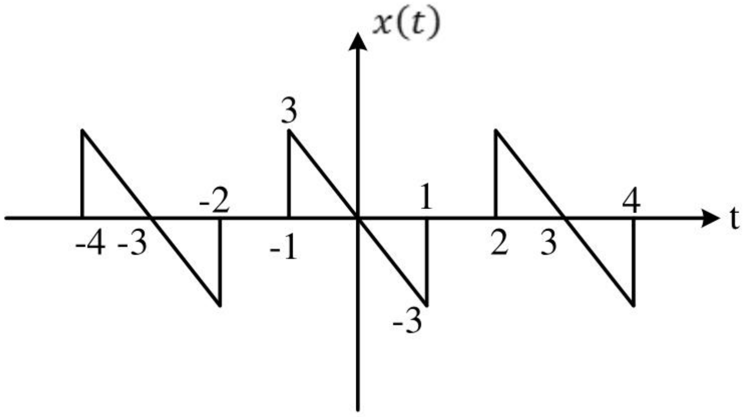
Thus the power in the n^{th} harmonic component is $\propto {}^{1}/_{n^2}$

Ratio of power in 7^{th} harmonic to 5^{th} harmonic for the given wage form is

$$\frac{{}^{1}/_{7^2}}{{}^{1}/_{5^2}} = \frac{25}{29} \cong 0.5$$

Answer 0.5

6. The waveform of a periodic signal $x(t)$ is shown in the figure.



A signal $g(t)$ is defined by $g(t) = x\left(\frac{t-1}{2}\right)$. The average power of $g(t)$ is _________ .

[GATE 2015: 1 Mark]

Soln. The equation for the given waveform can be written as

$$x = -3\,t$$

The period of the waveform is 3 (i.e. from -1 to +2)

$$Av.Power = \frac{1}{T}\int [x(t)]^2\,dt$$

$$= \frac{1}{3}\left[\int_{-1}^{0} (-3\,t)^2\,dt + \int_{0}^{1} (-3\,t)^2\,dt + \int_{1}^{2} 0^2.dt\right]$$

$$= \frac{1}{3}\left[9.\frac{t^3}{3}\,\Big|_{-1}^{0} + 9\,\frac{t^3}{3}\,\Big|_{0}^{1} + 0\right]$$

$$= \frac{1}{3}\left[\frac{9}{3}.\{0-(-1)\} + \frac{9}{3}(1-0)\right]$$

$$= \frac{1}{3}\left[\frac{9}{3} + \frac{9}{3}\right] = \frac{1}{3} \times \frac{18}{3} = 2$$

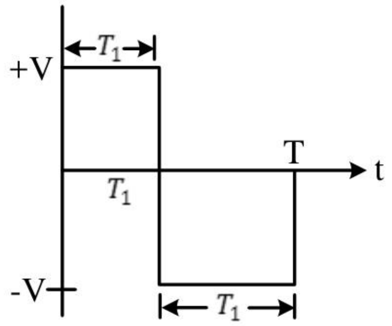
Answer 2

7. The RMS value of a rectangular wave of period T, having a value of +V for a duration T_1 $(< T)$ and $-V$ for the duration $T - T_1 = T_2$, equals

(a) V
(b) $\frac{T_1 - T_2}{T} V$
(c) $\frac{V}{\sqrt{2}}$
(d) $\frac{T_1}{T_2} V$

[GATE: 1995 1 Mark]

Soln.



The waveform can be drawn as per the given problem.

Period $(T) = T_1 + T_2$

$$RMS\ value = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

$$-\overline{\left[\int_0^{T_1} V^2\,dt + \int_{T_1}^{T} (-V)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{T}[V^2.(T_1 - 0) + V^2(T - T_1)]}$$

$$= \sqrt{\frac{1}{T}.V^2\,[T_1 + T - T_1]} = \sqrt{V^2} = V \qquad Option\ (a)$$

7. The RMS value of a rectangular wave of period T, having a value of +V for a duration $T_1\ (< T)\ and\ -V$ for the duration $T - T_1 = T_2$, equals

(a) V

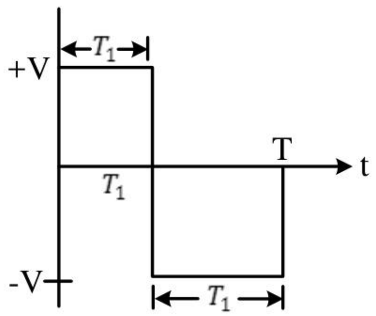
(b) $\frac{T_1 - T_2}{T} V$

(c) $\frac{V}{\sqrt{2}}$

(d) $\frac{T_1}{T_2} V$

[GATE: 1995 1 Mark]

Soln.



The waveform can be drawn as per the given problem.

Period $(T) = T_1 + T_2$

$$RMS\ value = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

$$= \sqrt{\left[\int_0^{T_1} V^2\,dt + \int_{T_1}^{T} (-V)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{T}[V^2.(T_1 - 0) + V^2(T - T_1)]}$$

$$= \sqrt{\frac{1}{T}.V^2\,[\,T_1 + T - T_1]} = \sqrt{V^2} = V \qquad Option\ (a)$$

Fourier series

8. The trigonometric Fourier series of an even function of time does not have
   (a) the dc term
   (b) cosine terms
   (c) sine terms
   (d) odd harmonic terms

[GATE 1996: 1 Mark]

Soln. For periodic even function, the trigonometric Fourier series does not contain the sine terms (odd functions)

It has dc term and cosine terms of all harmonics.

Option (c)

9. The trigonometric Fourier series of a periodic time function can have only
   (a) cosine terms
   (b) sine terms
   (c) cosine and sine terms
   (d) dc and cosine terms

Soln. The Fourier series of a periodic function $x(t)$ is given by the form

$$x(t) = \sum_{n-0}^{\infty} a_n \cos n\, \omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\, \omega_0\, t$$

Thus the series has cosine terms of all harmonics: $n\omega_0, n = 0,1,2 - - -$

Where 0^{th} harmonic = dc term (average or mean) = a_0 and sine terms of all harmonics: $n\omega_0, n = 1,2, - - - -$.

10. The Fourier series of an odd periodic function, contains only
    (a) odd harmonics
    (b) even harmonics
    (c) cosine terms
    (d) sine terms

[GATE 1994: 1 Mark]

Soln. If periodic function is odd the dc term $a_0 = 0$ and also cosine terms (even symmetry)

It contains only sine terms

Option (d)

11. The Fourier series of a real periodic function has only
    P. Cosine terms if it is even
    Q. Sine terms if it is even
    R. Cosine terms if it is odd
    S. Sine terms if it odd

    Which of the above statements are correct?

(a) P and S
(b) P and R
(c) Q and S
(d) Q and R

[GATE 2009: 1 Mark]

Soln. The Fourier series for a real periodic function has only cosine terms if it is even and sine terms if it is odd

Option (a)

12. The trigonometric Fourier series of an even function does not have the
(a) dc term
(b) cosine terms
(c) sine terms
(d) odd harmonic terms

Soln. The trigonometric Fourier series of an even function has cosine terms which are even functions.

It has dc term if its average value is finite and no dc term if average value is zero

So it does not have sine terms

Option (c)

13. Which of the following cannot be the Fourier series expansion of a periodic signals?

(a) $x(t) = 2\cos t + 3\cos 3t$

(b) $x(t) = 2\cos \pi t + 7\cos t$

(c) $x(t) = \cos t + 0.5$

(d) $x(t) 2\cos 1.5\pi t + \sin 3.5\pi t$

[GATE 2002: 1 Mark]

Soln. (a) $x(t) = 2\cos t + 3\cos t$ is periodic signal with fundamental frequency $\omega_0 = 1$

(b) $x(t) = 2\cos \pi\, t + 7\cos t$ The frequency of first term $\omega_1 = \pi$ frequency of 2nd term is $\omega_2 = 1$

$$\frac{\omega_1}{\omega_2} = \frac{\pi}{1} \; is\ not\ the\ rational\ number$$

So $x(t)$ is aperiodic or not periodic

(c) $x(t) = \cos t + 0.5$ is a periodic function with $\omega_0 = 1$

(d) $x(t) = 2\cos(1.5\pi)t + \sin(3.5\pi)t$ first term has frequency $\omega_1 = 1.5\pi$ 2nd term has frequency $\omega_2 = 3.5\pi$

$$\frac{\omega_1}{\omega_2} = \frac{1.5\pi}{3.5\pi} = \frac{1.5}{3.5} = \frac{3 \times 0.5}{7 \times 0.5} = \frac{3}{7}$$

So about ratio is rational number $x(t)$ is a periodic signal, with fundamental frequency $\omega_0 = 0.5\pi$

Since function in (b) is non periodic. So does not satisfy Dirictilet condition and cannot be expanded in Fourier series

14. Choose the function $f(t), -\infty < t < \infty$, for which a Fourier series cannot be defined.
    (a) $3\sin(25t)$
    (b) $4\cos(20t+3)+2\sin(710t)$
    (c) $\exp(-|t|)\sin(25t)$
    (d) 1

Soln. Fourier series is defined for periodic function and constant

(a) $3\sin(25\,t)$ *is periodic* $\omega = 25$
(b) $4\cos(20\,t+3)+2\sin(710\,t)$ sum of two periodic function is also periodic function
(c) $e^{-|t|}\sin 25\,t$ Due to decaying exponential decaying function it is not periodic. So Fourier series cannot be defined for it.
(d) Constant, Fourier series exists.
Fourier series can't be defined for option (c)

15. A periodic signal $x(t) of\ period\ T_0$ is given by

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, T_1 < |t| < \dfrac{T_0}{2} \end{cases}$$

The dc component of $x(t)$ is

(a) $\frac{T_1}{T_0}$

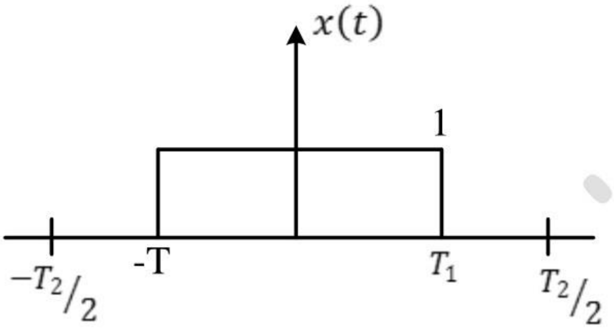
(b) $\frac{T_1}{(2T_0)}$

(c) $\frac{2T_1}{T_0}$

(d) $\frac{T_0}{T_1}$

[GATE 1998: 1 Mark]

Soln.



Given periodic signal can be drawn having period T_0

Fourier series the function $x(t)$ can be written as

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_o t + b_n \sin n\omega_0 t$$

Where dc component given by

$$a_0 = \frac{1}{T_0} \int_{T_0}^{\infty} x(t)\, dt$$

$$a_0 = \frac{1}{T} \int_{-T_0/2}^{T_0/2} x(t) dt$$

$$= \frac{1}{T_0}\left[\int_{-T_0/2}^{-T_1} x(t)dt + \int_{-T_1}^{T_1} x(t)dt + \int_{T_1}^{T_0/2} x(t)dt \right]$$

$$= \frac{1}{T_0}[0 + 2T_1 + 0]$$

$$= \frac{2T_1}{T_0}$$ Option (c)

16. The Fourier series representation of an impulse train denoted by

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) \ \ is\ given\ by$$

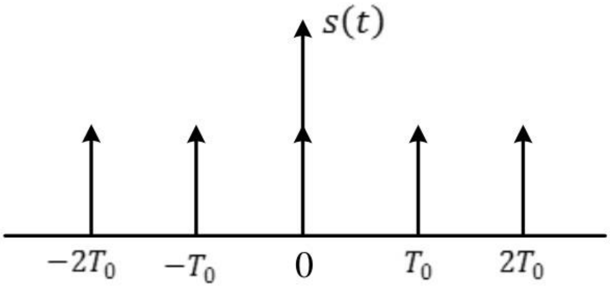
$$(a) \quad \left(\frac{1}{T_0}\right) \sum_{n=-\infty}^{\infty} \exp(-j2\pi n\, t/T_0)$$

$$(b) \quad \left(\frac{1}{T_0}\right) \sum_{n=-\infty}^{\infty} \exp(-j\pi n\, t/T_0)$$

$$(c) \quad \left(\frac{1}{T_0}\right) \sum_{n=-\infty}^{\infty} \exp(j\pi n\, t/T_0)$$

$$(d) \quad \left(\frac{1}{T_0}\right) \sum_{n=-\infty}^{\infty} \exp(j2\pi n\, t/T_0)$$

Soln.



The given impulse train $s(t)$ with strength of each impulse as 1 is aperiodic function with period T_0

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad where\ \omega_0 = \frac{2T}{T_0}$$

$$where\ C_n = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} \delta(t) e^{-jn\omega_0 t}\, dt \ = \frac{1.\, e^{-jn\omega_0 t}}{T_0}\Big|t = 0 \quad = {}^{1}/_{T_0}$$

17. The Fourier series expansion of a real periodic signal with fundamental frequency f_0 is given by

$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{j\,2\pi n\, f_0\, t}$$

It is given that $C_3 = 3 + j5 \quad then \quad C_{-3} \;\; is$

(a) $5 + 3j$ (c) $-5 + 3j$

(b) $-3 - j5$ (d) $3 - j5$

Soln. Given $C_3 = 3 + j5$

We know that for real periodic signal

$$C_{-k} = C_k^*$$

$$So, \quad C_{-3} = C_3^* = (3 - j5)$$

Continuous Time signal – Fourier Transform

1. The Fourier transform of a real valued time signal has
   (a) odd symmetry
   (b) even symmetry
   (c) conjugate symmetry
   (d) no symmetry

[GATE 1996: 1 Mark]

Soln. For real valued time signal, Fourier Transform has conjugate symmetry.

If $x(t)$ $is\ real$ $\rightarrow Fourier\ Transform$ is $X(f)$

Then there exists conjugate even symmetry (Also called Hermitian Symmetry)

i.e. $X(f) = X^*(-f)$

or $X^*(f) = X(-f)$

From above condition it can be shown that

$|X(f)|\ and\ R_e\{X(f)\}$ have even symmetry

i.e. $|X(f)| = |X(-f)|$

$\angle X(f) and\ I_m\{X(f)\}$ have odd symmetry

$\angle X(f) = -\angle X(-f)$

Option (c)

2. A signal x(t) has a Fourier transform X(ω). If x(t) is a real and odd function of t, them X(ω) is
   (a) a real and even function of ω
   (b) an imaginary and odd function of ω
   (c) an imaginary and even function of ω
   (d) a real and odd function of ω

[GATE 1999: 1 Mark]

Soln. If $f(t)$ is real and even then $F(\omega)$ is real

Even → $f(t) = f(-t)$

$$F(\omega) = F(-\omega)$$

Real → $f(-\omega) = f^*(\omega)$

Or $F(\omega) = F^*(\omega)$

If $f(t)$ is real and odd

$F(\omega)$ is pure imaginary

odd → $f(t) = -f(-t)$

$$F(\omega) = -F(-\omega)$$

Option (b)

3. The Fourier transform of a conjugate symmetric function is always

(a) imaginary
(b) conjugate anti-symmetric
(c) real
(d) conjugate symmetric

[GATE 2004: 1 Mark]

Soln. Given that the time function $x(t)$ is conjugate symmetric i.e.

If $x(t) = x^*(-t)$

Use the property of conjugate symmetry of FT

If $x(t) \rightarrow X(f)$

Then $x^*(-t) = X^*(f)$

Given $x(t) = x^*(-t)$

Then $X(f) = X^*(f)$

So, $X(f)$ is real

Option (c)

4. If $G(f)$ represents the Fourier Transform of a signal $g(t)$ which is real and odd symmetric in time, then

(a) G(f) is complex
(b) G(f) is imaginary
(c) G(f) is real
(d) G(f) is real

[GATE 1992: 2 Marks]

Soln. $g(t) \rightarrow G(f)$

Note, If $g(t)$ is real and even,

$G(f)$ is also real and even

But if $g(t)$ is real and odd

$G(f)$ is imaginary and odd

Option (b)

5. The amplitude spectrum of a Gaussian pulse is
   (a) uniform
   (b) a sine function
   (c) Gaussian
   (d) An impulse function

[GATE 1998: 1 Mark]

Soln. Gaussian pulse is defined by

$$f(t) = e^{-\pi t^2}$$

Fourier Transform of this pulse can be evaluated

$$\mathcal{F}\left[e^{-\pi t^2}\right] = \int_{-\infty}^{\infty} e^{-\pi t^2} \; e^{-j\omega t}\, dt$$

After evaluation of integral one gets

$$\mathcal{F}\left[e^{-\pi t^2}\right] = e^{-\pi f^2}$$

When area under Gaussian pulse and central ordinate of the pulse is unity, it is said to be normalized Gaussian pulse. Such pulse is its own Fourier Transform

$$e^{-\pi t^2} \longleftrightarrow e^{-\pi f^2}$$

Option (c)

6. The Fourier Transform of the signal $x(t) = e^{-3t^2}$ is of the following from where A and B are constants:

(a) $A\,e^{-B|f|}$

(b) $A\,e^{-Bf}$

(c) $A + B|f|^2$

(d) $A\,e^{-Bf^2}$

[GATE 2000: 1 Mark]

Soln. The Fourier Transform of a normalized Gaussian pulse is also normalized Gaussian pulse

For $g(t) = e^{-at^2}$

$$G(\omega) = \sqrt{\frac{\pi}{a}}\, e^{-\omega^2/4a}$$

So, it is of the form

$$Ae^{-Bf^2}$$

The constants A and B can be found

For $x(t) = e^{-3t^2}$

Here a = 3

So, $X(\omega) = \sqrt{\frac{\pi}{3}}\,.\,e^{-\omega^2/4\times3}$

$$X(\omega) = \sqrt{\frac{\pi}{3}}\,.\;e^{-\omega^2/12}$$

Option (d)

7. The function f(t) has Fourier Transform g(ω). The Fourier Transform of

$$g(t) = \left(\int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt \right) \; is$$

(a) $\frac{1}{2\pi} f(\omega)$

(b) $\frac{1}{2\pi} f(-\omega)$

(c) $2\pi f(-\omega)$

(d) $none \; of \; above$

[GATE 1997: 1 Mark]

Soln. Given

$$f(t) \longleftrightarrow g(\omega)$$

Then F [g(t)] ?

Inverse transform

$$f(t) = \frac{1}{2} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$Or, \;\; 2\pi f(-t) = \int_{-\infty}^{\infty} F(\omega)\; e^{j\omega t}\, d\omega$$

Here ω is dummy variable so can be exchanged

$$i.e \;\; 2\pi f(-\omega) = \int_{-\infty}^{\infty} F(t) e^{-j\omega t}\, dt = \mathcal{F}[f(t)]$$

Above equation shows that Fourier transform of time function f(t) is $2\pi f(-\omega)$

In this problem also, $g(\omega)$ is Fourier Transform for $f(t)$

So changing dummy variable (from t to ω) then $F\{g(t)\} = 2\pi f(-\omega)$

Option (c)

8. The Fourier transform of a function x(t) is X(f). The Fourier transform of $\frac{dx(t)}{dt}$ will be

(a) $\frac{dX(f)}{dt}$

(b) $j2\pi f\ X(f)$

(c) $jf\ X(f)$

(d) $\frac{X(f)}{jf}$

[GATE 1998: 1 Mark]

Soln.

$$If\ x(t) \longleftrightarrow X(f)$$

$$then\ \frac{dx}{dt} \longleftrightarrow j\omega\, X(\omega)$$

$$Since,\quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\ d\omega$$

Then

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}.\frac{d}{dt}\left[\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\ d\omega\right]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d}{dt}\{X(\omega)e^{j\omega t}\}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega\, X(\omega)\, e^{j\omega t}\, d\omega$$

$$= \mathcal{F}^{-1}[j\omega\, X(\omega)]$$

$$= \mathcal{F}^{-1}[j2\pi\ \ X(f)]$$

This shows that differentiation in time domain is equivalent to multiplication by $j\omega = j2\pi f$ in frequency domain

Option (b)

9. The Fourier transform of a voltage signal x(t) is X(f). The unit of $|X(f)|$is
   (a) Volt (c) Volt / sec
   (b) Volt – sec (d) $Volt^2$

[GATE 1998: 1 Mark]

Soln. As per the definition of Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t} . dt$$

Looking at R.H.S expression, then unit of X(f) will be volt – sec

Option (b)

10. If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to
    (a) E (c) 2E
    (b) E/2 (d) 4E

[GATE 2001: 1 Mark]

Soln. Given,

Signal $f(t)$ has energy E.

Find energy of the signal $f(2t)$.

Energy of signal

$$f(t) = \int_{-\infty}^{\infty} f^2(t) dt$$

So, energy of signal $f(2t)$ will be

$$= \int_{-\infty}^{\infty} f^2(2t)\ dt$$

$$= \int_{-\infty}^{\infty} f^2(\tau)\ \frac{d\tau}{2} = \frac{E}{2}$$

Option (b)

11. The Fourier transform $F\{e^{-t}\,u(t)\}$ is equal to $\frac{1}{1+j2\pi f}$ $Therefore,$ $F\left\{\frac{1}{1+j2\pi t}\right\}$ is

(a) $e^{f}u(f)$
(b) $e^{-f}u(f)$
(c) $e^{f}u(-f)$
(d) $e^{-f}u(-f)$

[GATE 2002: 1 Mark]

Soln. Given,

$$\mathcal{F}[e^{-t}\,u(t)] = \frac{1}{(1+j2\pi f)}$$

Using the duality property

If $g(t) \rightarrow G(f)$

$G(t) \rightarrow g(-f)$

Therefore,

$$\frac{1}{(1+j2\pi t)} \rightarrow e^{f}u(-f)$$

Option (c)

12. Let $x(t) \leftrightarrow X(j\omega)$ be Fourier Transform pair. The Fourier Transform of the signal $x(5t-3)$ in terms of $X(j\omega)$ is given as

(a) $\frac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

(b) $\frac{1}{5}e^{\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

(c) $\frac{1}{5}e^{-j3\omega}X\left(\frac{j\omega}{5}\right)$

(d) $\frac{1}{5}e^{j3\omega}X\left(\frac{j\omega}{5}\right)$

[GATE 2006: 1 Mark]

Soln. Given,

$$x(t) \longleftrightarrow X(j\omega)$$

Find Fourier Transform of $x(5t-3)$

Time shifting property

$$x(t \mp t_0) \longrightarrow e^{\pm j t_0 \omega} X(j\omega)$$

Scaling property

$$x(Kt) \longrightarrow \frac{1}{|K|} X\left(j\frac{\omega}{K}\right)$$

Using time shifting property

$$x(t-3) \longrightarrow e^{-j3\omega} . X(j\omega)$$

Using scaling property

$$x(5t-3) = \frac{1}{5}e^{-j\frac{3\omega}{5}} \times \left(\frac{j\omega}{5}\right)$$

Option (a)

13. If the Fourier Transform of a deterministic signal g(t) is G(f), then

Items – 1

(1) The Fourier Transform of $g(t-2)$ is

(2) The Fourier Transform of $g(t/2)$ is

Items – 2

(A) $G(f)e^{-j\,(4\pi f)}$

(B) G(2f)

(C) 2G(2f)

(D) G(f - 2)

Match each of the items 1, 2 on the left with the most appropriate item A, B, C, or D on the right.

[GATE 1997: 2 Marks]

Soln. $g(t) \;\leftrightarrow\; G(f)$

$$g(t-2) \;\leftrightarrow\; e^{-j2\pi 2f}\,G(f) = G(f)e^{-j(4\pi f)}$$

$$g\left(\frac{t}{2}\right) \;\leftrightarrow\; \left(\frac{1}{1/2}\right) G\left(\frac{f}{1/2}\right) = 2G(2f)$$

Option 1 – A, 2 – C

14. Let x(t) and y(t) (with Fourier transform X(f) and Y(f) respectively) be related as shown in Figure (1) & (2).

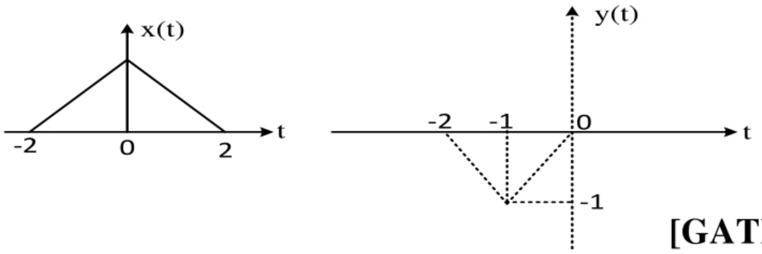
Then $Y(f)$ is

(a) $-\frac{1}{2}X(f/2)e^{-j2\pi f}$

(b) $-\frac{1}{2}X(f/2)e^{j2\pi f}$

(c) $-X(f/2)e^{j2\pi f}$

(d) $-X(f/2)e^{-j2\pi f}$



[GATE 2004: 2 Marks]

Soln. The figures of $x(t)$ and $y(t)$ are given, from these figures.

$$y(t) = -x(2t+2)$$

$$If\ x(t) \leftrightarrow X(f)$$

$$Then\ \ x(t+2) \rightarrow e^{j2\pi 2f}\, X(f)$$

Using time shifting property

$$x(2t+2) \rightarrow \left(\frac{1}{2}\right)X(f/2)e^{j2\pi f}$$

According to time scaling property

$$y(f) = -\frac{1}{2}X(f/2)e^{j2\pi f}$$

Option (b)

15.For a signal x(t) the Fourier transform is X(f). Then the inverse Fourier transform of X(3f + 2) is given by

(a) $\frac{1}{2}x\left(\frac{t}{2}\right)e^{j3\pi t}$

(b) $\frac{1}{3}x\left(\frac{t}{3}\right)e^{-j4\pi/3}$

(c) $3x(3t)e^{-j\,4\pi t}$

(d) $x(3t+2)$

[GATE 2005: 2 Marks]

Soln. In this problem we use the following two properties of Fourier Transform

If $x(t) \rightarrow X(f)$

$$e^{\mp j2\pi f_0 t}\,x(t) \longrightarrow X(f \pm f_0) \quad ----(1)$$

Frequency shifting property

$$\frac{1}{|k|}x\left(\frac{t}{k}\right) \longrightarrow (kf) \quad ----------(2)$$

Time scaling property

Using frequency shift property

$$e^{-j4\pi t}x(t) \longrightarrow X(f+2)$$

Using time scaling property

$$\frac{1}{3}x\left(\frac{t}{3}\right)e^{-4\pi t/3} \longrightarrow X(3f+2)$$

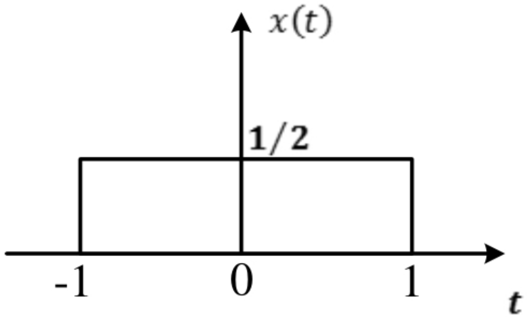
Option (b)

16. Two of the angular frequencies at which its Fourier transform becomes zero are

(a) $\pi, 2\pi$
(b) $0.5\ \pi, 1.5\ \pi$
(c) $0, \pi$
(d) $2\ \pi, 2.5\ \pi$

[GATE 2008: 2 Marks]

Soln. The given time function $x(t)$ is shown is figure



Its Fourier Transform $X(f)$ is given by

$$X(f) = 2 \sin c(2f)$$

$$= 2\ for\ f = 0$$

$$= 0\ for\ 2f = \pm 1,\ \pm 2, - - - -$$

Or $\omega = 2\pi f = \pm\pi,\ \pm 2\pi,\ \pm 3\pi, - - - - -$

Option (a)

17. The Fourier transform of a signal h(t) is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of h(0) is

(a) 1/4
(b) 1/2
(c) 1
(d) 2

[GATE 2012: 2 Marks]

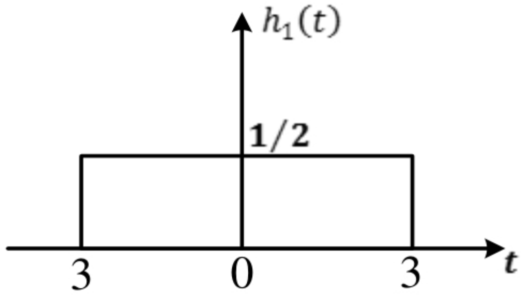
Soln.

$$H(j\omega) = \frac{(2\cos\omega)(\sin 2\omega)}{\omega}$$

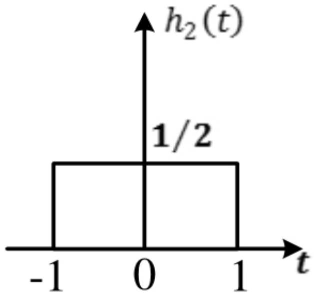
$$= \frac{2\sin 2\omega\ .\ \cos\omega}{\omega}$$

$$= \frac{\sin 3\omega + \sin\omega}{\omega}$$

$$= \frac{\sin 3\omega}{\omega} + \frac{\sin\omega}{\omega}$$



$\longleftrightarrow \quad \dfrac{\sin 3\omega}{\omega}$



$\longleftrightarrow \quad \dfrac{\sin\omega}{\omega}$

So, inverse Fourier Transform of $H(j\omega)$

$$h(t) = h_1(t) + h_2(t)$$

$$h(0) = h_1(0) + h_2(0) = \frac{1}{2} + \frac{1}{2} = 1$$

Option (c)

18. Let $g(t) = e^{-\pi t^2}$, and $h(t)$ is filter marched to $g(t)$. If $g(t)$ is applied as input to $h(t)$, then the Fourier transform of the output is

(a) $e^{-\pi t^2}$

(b) $e^{-\pi f^2/2}$

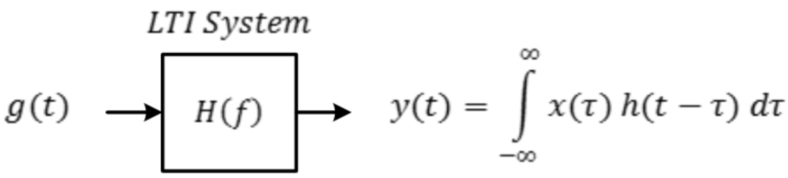
(c) $e^{-\pi|f|}$

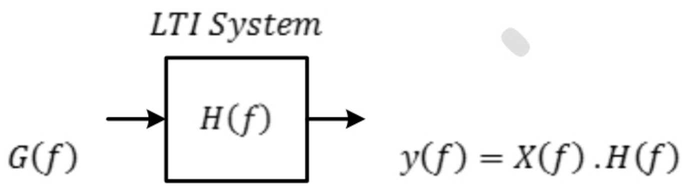
(d) $e^{-2\pi f^2}$

[GATE 2013: 1 Mark]

Soln. Given, $g(t) = e^{-\pi t^2}$

$h(t)$ *is matched to* $g(t)$





$$g(t) = e^{-\pi t^2} \quad (Gaussian\ Pulse)$$

$$G(f) = e^{-\pi f^2} \quad (Fourier\ Transform\ of\ Gaussian\ Pulse)$$

$$h(f) = e^{-\pi f^2} \quad (Since\ filter\ is\ matched)$$

$$y(f) = G(f)\,.h(f) = e^{-\pi f^2}\ .\ e^{-\pi f^2}$$

$$y(f) = e^{-2\pi f^2}$$

Option (d)

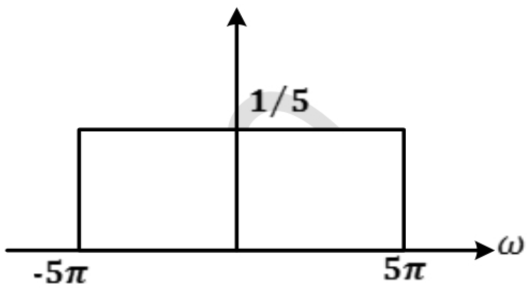
19. The value of the integral

$$\int_{-\infty}^{\infty} \sin c^2(dt) \quad is \_\_\_\_\_\_\_\_.$$

[GATE 2014: 1 Mark]

Soln. The given integral gives the energy of the signal $\sin c\ (5t)$

$$\sin c\ (5t) = \frac{\sin 5\pi t}{5\pi t} \longleftrightarrow$$



Using Perceval's theorem

$$E_f = \int_{-\infty}^{\infty} |f(t)|^2\ dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2\ d\omega$$

$$Energy = \frac{1}{2\pi} \int_{-5\pi}^{5\pi} (1/5)^2\ d\omega$$

$$= \frac{1}{50\pi}(10\pi) = \frac{1}{5} = 0.2$$

Answer 0.2

DTFT, DFT and FFT

1. Let $x(n) = \left(\frac{1}{2}\right)^n u(n), y(n) = x^2(n)\ and\ Y\left(e^{j\omega}\right)$ be the Fourier Transform of $y(n)$. Then $y(e^{j0})$ is

(a) $\frac{1}{4}$

(b) 2

(c) 4

(d) $\frac{4}{3}$

[GATE 2005: 1 Mark]

Soln. Given

$$\boldsymbol{x(n) = \left(\frac{1}{2}\right)^n u(n)}$$

And $\boldsymbol{y(n) = x^2(n) = \left(\frac{1}{2}\right)^{2n} u^2(n)}$

Or, $\boldsymbol{y(n) = \left[\left(\frac{1}{2}\right)^2\right]^n u(n) = \left(\frac{1}{4}\right)^n u(n)}$

Taking Fourier Transform

$$\boldsymbol{Y\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}}$$

$$\boldsymbol{= \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}}$$

$$\boldsymbol{So,\quad Y\left(e^{j0}\right) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + ----}$$

$$\boldsymbol{= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}}$$

Option (d)

2. A signal $x(n) = \sin(\omega_0 n + f)$ is the input to a linear time-invariant system having a frequency response $H(e^{j\omega})$. If the output of the system $Ax(n - n_0)$, then the most general form of $\angle H(e^{j\omega})$ will be
(a) $-n_0\omega_0 + \beta$ for any arbitrary real β
(b) $-n_0\omega_0 + 2\pi k$ for any arbitrary integer k
(c) $n_0\omega_0 + 2\pi k$ for any arbitrary integer k
(d) $-n_0\omega_0\phi$

[GATE 2005 : 2 Marks]

Soln. Given

$$y(n) = A\ x(n - n_0)$$

Taking Fourier Transform

$$Y(e^{j\omega}) = A\, e^{-j\omega_0 n_0}\ X(e^{j\omega})$$

Or, $$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = A\, e^{-j\omega_0 n_0}$$

Thus $\angle H(e^{j\omega}) = -\omega_0 n_0$

For LTI discrete time system phase and frequency of $H(e^{j\omega})$ are periodic with period 2π. So in general form

$$\theta(\omega) = -n_0\omega_0 + 2\pi k$$

Option (b)

3. A 5-point sequence $x[n]$ is given as

$x[-3] = 1,\ x[-2] = 1,\ x[-1] = 0,\ x[0] = 5, x[1] = 1$

Let $X(e^{j\omega})$ denote the discrete-time Fourier Transform of $x[n]$. The value of

$$\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega \quad is$$

(a) 5
(b) 10 π
(c) 16 π
(d) $5 + j\ 10\pi$

[GATE 2007 : 2 Marks]

Soln. Discrete Fourier Transform (DTFT) when $N \to \infty$ is given by

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

For n = 0, we get

$$x[0] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega .0}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$$

$$or, \quad \int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega = 2\pi\, x[0] = 2\pi \times 5$$

Option (b)

4. $\{x(n)\}$ is real-valued periodic sequence with a period N. $x(n)$ and $X(k)$ form N-point Discrete Fourier Transform (DFT) pairs. The DFT Y(k) of the sequence

$$y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\ X(n+r)\ is$$

(a) $|X(k)|^2$

(b) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X^*\ (k+r)$

(c) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X\ (k+r)$

(d) 0

[GATE 2008 : 2 Marks]

Soln. Given

$$\boldsymbol{y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\ x(n+r)}$$

$y(n)$ is the correlation of a signal $x(n)$ with itself.

The Fourier Transform of auto correlation function is $|X(k)|^2$

Option (b)

5. The 4-point discrete Fourier Transform (DFT) of a discrete time sequence $\{1, 0, 2, 3\}$ is

(a) $[0, -2+2j, 2, -2-2j]$
(b) $[2,\ 2+2j,\ 6,\ 2-2j]$
(c) $[6,\ 1-3j, 2,\ 1+3j]$
(d) $[6, -1+3j,\ 0, -1-3j]$

[GATE 2009 : 2 Marks]

Soln. Given discrete time sequence

$$x[n] = \{1, 0, 2, 3\}\ and\ N = 4$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\ e^{-j2\pi nk/N} \qquad k = 0, 1, --(N-1)$$

For N = 4

$$X[k] = \sum_{n=0}^{3} x[n]\ e^{-j2\pi nk/4} \qquad k = 0, 1, --3$$

Now

$$X[k] = \sum_{n=0}^{3} x[n] = x[0] + x[1] + x[2] + x[3]$$

$$= 1 + 0 + 2 + 3 = 6$$

$$x[1] = \sum_{n=0}^{3} x[n]\, e^{-\frac{j\pi n}{2}} = x[0] + x[1]e^{-\frac{j\pi}{2}} + x[2]e^{-j\pi} + x[3]e^{-j\pi 3/2}$$

$$= 1 + 0 - 2 + i3 = -1 + i3$$

$$x[2] = \sum_{n=0}^{3} x[n]\, e^{-j\pi n} = x[0] + x[1]e^{-j\pi} + x[2]e^{-2j\pi} + x[3]e^{-j3\pi}$$

$$= 1 + 0 + 2 - 3 = 0$$

$$x[3] = \sum_{n=0}^{3} x[n]\, e^{-j3\pi n/2} = x[0] + x[1]e^{-j3\pi/2} + x[2]e^{-j3\pi} + x[3]e^{-j3\pi/2}$$

$$= 1 + 0 - 2 - j3 = -1 - j3$$

Thus $[6, -1+j3, 0, -1-j3]$ **Option (d)**

6. For an N-point FFT algorithm with $N = 2^m$, which one of the following statements is TRUE?
   (a) It is not possible to construct is signal flow graph with both input and output in normal order
   (b) The number of butterflies in the mth state is N/m
   (c) In-place computation requires storage of only 2N node data
   (d) Computation of a butterfly requires only one complex multiplication

[GATE 2010 : 1 Mark]

Soln. For an N-point FFT algorithm.

Butterfly operate on one pair of samples and involves two complex additions and one complex multiplication

Option (d)

7. The first six points of the 8-point DFT of a real valued sequence are $5, 1 - j3, 0, 3 - 4j$, and $3 + j4$. The last two points of the DFT are respectively
   (a) $0, 1 - j3$
   (b) $0, 1 + j3$
   (c) $1 + j3, 5$
   (d) $1 - j3, 5$

Soln. Given that the sequence is real valued with 8 points.

i.e.

$$\boldsymbol{X(k) = [X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)]}$$

$$\boldsymbol{= [5, (1 - j3), 0, (3 - j4), 0, (3 + j4), -, -]}$$

If the sequence $x[n]$ is real then $X[k]$ is conjugate symmetric

i.e. $\boldsymbol{X[k] = X^*[N - k]}$

thus $\boldsymbol{X[6] = X^*[8 - 6] = X^*[2] = 0}$

$$\boldsymbol{X[7] = X^*[8 - 7] = X^*[1] = 1 + j3}$$

Option (b)

8. Consider a discrete time periodic signal $x[n] = \sin\left(\frac{\pi n}{5}\right)$. Let a_k be the complex. Fourier series coefficients of $x[n]$. The coefficients $\{a_k\}$ are non-zero when $k = Bm \pm 1$, where m is any integer. The value of B is ______.

[GATE 2014 : 2 Marks]

Soln. Given

$$x(n) = \sin\left[\frac{\pi n}{5}\right]$$

Find the value of B

$$x(n) = \sin\left[\frac{\pi n}{5}\right]$$

Time period of $x(n) = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi/5} = 10$

$$\sin\left[\frac{\pi}{5}n\right] = \frac{1}{2j}.e^{j\frac{\pi}{5}n} - \frac{1}{2j}\,e^{-j\frac{\pi}{5}n}$$

So, α_k **exist for** $K = \pm 1$

$x[n]$ **is a discrete time signal its Fourier series coefficients exist after each time interval value**

$i.e.$ $K = \pm 1, 10 \pm 1, 20 \pm 1, - - - -$

$$\alpha_k = 10m \pm 1 = Bm \pm 1$$

$i.e$ $B = 10$

Answer: B = 10

Z- Transform

1. The z – transform of the time function

$$\sum_{k=0}^{\infty} \delta(n-k) \ is$$

(a) $(z-1)/z$ (c) $z/(z-1)$

(b) $z/(z-1)^2$ (d) $(z-1)^2/z$

[GATE 1998: 1 Mark]

Soln. Time function is given

$$\boldsymbol{x(n) = \sum_{k=0}^{\infty} \delta\,(n-k)}$$

$$\boldsymbol{= \delta(n) + \delta(n-1) + \delta(n-2) + ---}$$

$$\boldsymbol{x(n) = u(n)}$$

$$\boldsymbol{x(z) = \mathcal{Z}[u(n)] = \frac{z}{(z-1)}}$$

Option (c)

2. The z – transform F(z) of the function $f(nT) = a^{nT}$ is

(a) $\frac{z}{z-a^T}$ (c) $\frac{z}{z-a^{-T}}$

(b) $\frac{z}{z+a^T}$ (d) $\frac{z}{z+a^{-T}}$

[GATE 1999: 1 Mark]

Soln. The z – transform is given by

$$\boldsymbol{\mathcal{Z}[f(nt)] = \sum_{n=-\infty}^{\infty} f(nT)\, z^{-n}}$$

$$\boldsymbol{= \sum_{n=-\infty}^{\infty} a^{nT}\, z^{-n}}$$

$$\boldsymbol{= \sum_{n=-\infty}^{\infty} (a^T z^{-1})^n}$$

$$\boldsymbol{f(z) = \frac{1}{1-a^T z^{-1}} = \frac{z}{z-a^T}}$$

Option (a)

3. The region of convergence of the z – transform of a unit step function is
   (a) $|z| > 1$
   (b) $|z| < 1$
   (c) (Real part of z) > 0
   (d) (Real part of z) < 0

[GATE 2001: 1 Mark]

Soln. **Given**

$$x(n) = u(n)$$

$$H(z) = \sum_{n=0}^{\infty} u(n)\, z^{-n} = \sum_{n=0}^{\infty} 1.\, z^{-n}$$

$$= \sum_{n=0}^{\infty} (z^{-1})^{n}$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + - - -$$

$$= \frac{1}{1 - 1/z} \quad where \quad \frac{1}{|z|} < 1$$

So, ROC is the range of value of z for which $|z| > 1$

Option (a)

4. A sequence $x(n)$ with the z – transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear time – variant system with the impulse response $h(n) = 2\delta(n-3)$ where

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & otherwise \end{cases}$$

The output at n = 4 is

(a) – 6 (c) 2

(b) zero (d) – 4

[GATE 2003: 1 Mark]

Soln. Given

$$\boldsymbol{X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}}$$

$$\boldsymbol{h(n) = 2\delta(n-3)}$$

$$\boldsymbol{h(z) = 2\sum_{n=0}^{\infty} \delta(n-3)\, z^{-n}}$$

$$\boldsymbol{= 2.z^{-3}}$$

$$\boldsymbol{Note, \qquad \mathcal{Z}[\delta(n-k)] = z^{-k}}$$

Also,

$$\boldsymbol{y(z) = H(z)\,.X(z)}$$

$$\boldsymbol{= 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})}$$

$$\boldsymbol{= 2(z + z^{-1} - 2z^{-2} + 2z^{-3} - 3z^{-7})}$$

Taking inverse z transform

$$\boldsymbol{y(n) = 2[\delta(n+1) + \delta(n-1) - 2\delta(n-2) + 2\delta(n-3) - 3\delta(n-7)]}$$

$$\boldsymbol{for\ n = 4}$$

$$\boldsymbol{y(4) = 2[\delta(5) + \delta(3) - 2\delta(2) + 2\delta(1) - 3\delta(-3)]}$$

At $\boldsymbol{n = 4, \quad y(4) = 0}$

Option (b)

5. The z – transform of a system is

$$H(z) = \frac{z}{z - 0.2}$$

If the ROC is $|z| < 0.2$, then the impulse response of the system is

(a) $(0.2)^n\, u[n]$ (c) $-(0.2)^2\, u[n]$

(b) $(0.2)^2\, u[-n-1]$ (d) $-(0.2)^n\, u[-n-1]$

Soln. Given

$$\boldsymbol{H(z) = \frac{z}{(z-0.2)}}$$

$$\boldsymbol{= \frac{z}{z(1-0.2z^{-1})}}$$

$$\boldsymbol{= \frac{1}{(1-0.2z^{-1})}}$$

Given ROC is $|z| < 0.2$

Comparing with

$$\boldsymbol{-a^n u(-n-1) \xleftrightarrow{\ Z\ } \frac{1}{(1 - az^{-1})}}$$

So, $h(n) = -(0.2)^n u(-n-1)$ Option (d)

6. The region of convergence of z – transform of the sequence

$$\left(\frac{5}{6}\right)^n u(n) - \left(\frac{5}{6}\right)^n u(-n-1)\ must\ be$$

(a) $|z| < \frac{5}{6}$

(b) $|z| > \frac{5}{6}$

(c) $\frac{5}{6} < |z| < \frac{5}{6}$

(d) $\frac{5}{6} < |z| < \infty$

[GATE 2005: 1 Mark]

Soln. For the given sequence we have to find ROC of z – transform.

Given sequence is

$$\boldsymbol{x(n) = \left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)}$$

$\boldsymbol{first\ term\ of\ the\ sequence\ is}$

$$\boldsymbol{x(n) = \left(\frac{5}{6}\right)^n u(n)\ \ which\ is\ right\ handed}$$

Sequence and is of the form $\boldsymbol{a^n u(n)}$

Its ROC extends outward.

$$\boldsymbol{ROC:\ |z| > \frac{5}{6}}$$

Second term is the left handed sequences

$\boldsymbol{-\left(\frac{6}{5}\right)^n u(-n-1)\ and\ is\ for\ the\ form - a^n u(-n-1)\ with\ ROC: < \frac{6}{5}}$

So, the combined ROC would be

$$\boldsymbol{\frac{5}{6} < |z| < \frac{6}{5}}$$

Option (c)

7. If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}$, then the region of convergences of $x_1[n] - x_2[n]$ includes

(a) $\frac{1}{3} < |z| < 3$

(b) $\frac{2}{3} < |z| < 3$

(c) $\frac{3}{2} < |z| < 3$

(d) $\frac{1}{3} < |z| < \frac{2}{3}$

[GATE 2006: 1 Mark]

Soln. Given

ROC of the given sequence

$$\boldsymbol{x_1[n] + x_2[n]}$$

$$\boldsymbol{is\ \frac{1}{3} < |z| < \frac{2}{3}\ then\ find\ \ ROC\ of\ \ x_1[n] - x_2[n]}$$

The ROC of addition or subtraction of two functions

$$\boldsymbol{x_1(n)\ and\ x_2(n)\ \ is\ \ R_1 \cap R_2\ \ .\ \ So\ same\ as\ above}$$

Option (d)

8. The ROC of z – transform of the discrete time sequence

$$x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)\ is$$

(a) $\frac{1}{3} < |z| < \frac{1}{2}$

(b) $|z| < \frac{1}{2}$

(c) $|z| < \frac{1}{3}$

(d) $2 < |z| < 3$

[GATE 2009: 1 Mark]

Soln. Given

$$\boldsymbol{x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)}$$

Taking z – transform

$$\boldsymbol{X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} - \sum_{n=-\infty}^{n=-1} \left(\frac{1}{2}\right)^n z^{-n}}$$

$$\boldsymbol{= \sum_{n=0}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n - \sum_{n=-\infty}^{-1} \left(\frac{1}{2} z^{-1}\right)^n}$$

First term gives ROC $\boldsymbol{\frac{1}{3} z^{-1} < 1 \quad or \quad |z| > \frac{1}{3}}$

Second term gives ROC $\boldsymbol{\frac{1}{2} z^{-1} > 1 \quad or \quad |z| < \frac{1}{2}}$

Thus the combined ROC is common ROC of both terms

$$\boldsymbol{\frac{1}{3} < |z| < \frac{1}{2}}$$

Option (a)

9. Consider the z – transform $X(z) = 5z^2 + 4z^{-1} + 3; 0 < |z| < \infty$. The inverse z – transform $x[n]$ is
   (a) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$
   (b) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$
   (c) $5u[n+2] + 3\,u[n] + 4\,u[n-1]$
   (d) $5u[n-2] + 3\,u[n] + 4\,u[n+1]$

[GATE 2010: 1 Mark]

Soln. Given

Z – transform $\quad X(z) = 5z^2 + 4z^{-1} + 3$

ROC : $0 < |z| < \infty$

To find inverse z – transform $x[n]$

We know,

$$\delta(n \pm a) \xleftrightarrow{Z} z^{\pm a}$$

$$\mathcal{Z}^{-1}[X(z)] = 5\delta[n+2] + 4\delta[n-1] + 3\delta[n]$$

Option (a)

10. Two discrete time systems with impulse responses $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is

(a) $\delta[n-1] + \delta[n-2]$

(b) $\delta[n-4]$

(c) $\delta[n-3]$

(d) $\delta[n-1]\ \delta[n-2]$

[GATE 2010: 1 Mark]

Soln. Given

$$h_1(n)\delta[n-1] \xleftrightarrow{Z} H_1(z) = z^{-1}$$

$$h_2(n)\delta[n-2] \xleftrightarrow{Z} H_2(z) = z^{-2}$$

Response of the cascaded system is

$$\boldsymbol{H(z) = H_1(z)\,.\,H_2(z)}$$

$$\boldsymbol{z^{-1}\,.\,z^{-2} = z^{-3}}$$

So, $\boldsymbol{h[n] = \delta[n-3]}$

Option (c)

11. If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its Z – transform in the Z – plane will be

(a) $\frac{1}{3} < |z| < 3$

(b) $\frac{1}{3} < |z| < \frac{1}{2}$

(c) $\frac{1}{2} < |z| < 3$

(d) $\frac{1}{3} < |z|$

[GATE 2012: 1 Mark]

Soln. Given

$$\boldsymbol{x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u(n)}$$

$$\boldsymbol{= \left(\frac{1}{3}\right)^n u(n) + \left(\frac{1}{3}\right)^{-n} u[-n-1] - \left(\frac{1}{2}\right)^n u[n]}$$

First term

$$\left(\frac{1}{3}\right)^n u(n) \longleftrightarrow \frac{1}{\left(1 - \frac{1}{3}z^{-1}\right)} \ : \ ROC\ |z| > \frac{1}{3}$$

Second term

$$\left(\frac{1}{3}\right)^{-n} u(-n-1) \longleftrightarrow \frac{1}{\left(1 - \frac{1}{3}z^{-1}\right)} \ : \ ROC\ |z| < 3$$

Third term

$$\left(\frac{1}{2}\right)^n u(n) \longleftrightarrow \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)} \ ; \ ROC\ |z| < \frac{1}{2}$$

Overall ROC will be interaction of these ROCs

$$\boldsymbol{\frac{1}{2} < |z| < 3}$$

Option (c)

12. Let $x[n] = x[-n]$. Let X(z) be the z – transform of $x[n]$. If $0.5 + j0.25$ is a zero of X(z), which one of the following must also be zero of X(z).

(a) $0.5 - j0.25$

(b) $\frac{1}{(0.5+j0.25)}$

(c) $\frac{1}{(0.5-j0.25)}$

(d) $2 + j4$

[GATE 2014: 1 Mark]

Soln. Given $\boldsymbol{x[n] = x[-n]}$

We know

$$x[n] \xleftrightarrow{Z} X[z]$$

$$x[-n] \xleftrightarrow{Z} X[z^{-1}]$$

Time reversal property in z – transform

So, if one zero is (0.5 + j 0.25). Then the other zero will

$$\boldsymbol{be} \quad \frac{\mathbf{1}}{(\mathbf{0.5} + \boldsymbol{j}\,\mathbf{0.25})}$$

Option (b)